closed-loop system is ESPR. Now using Matlab LMI Control Toolbox and solving the LMI (32), we obtain

$$X = \begin{bmatrix} 4.6705 & 3.6760 & -2.9312 \\ 3.6760 & 7.1592 & -3.9770 \\ -2.9312 & -3.9770 & 5.6329 \end{bmatrix}$$
$$Y = \begin{bmatrix} 1.6713 & -0.0105 & -0.8890 \\ -0.0105 & 0.4083 & 0.1350 \\ -0.8890 & 0.1350 & 0.8602 \end{bmatrix}$$
$$W = \begin{bmatrix} 1.0065 & 0.0081 & -0.1192 \\ 0.0081 & 0.0558 & -0.1015 \\ -0.1192 & -0.1015 & 0.8039 \end{bmatrix}$$
$$Z = \begin{bmatrix} 0.5721 & 1.4912 & -1.9891 \\ -0.1030 & 0.9699 & -1.9244 \end{bmatrix} \quad \epsilon = 3.4374.$$

Therefore, from Theorem 3, there exists a solution to the positive real control problem. Furthermore, a desired state feedback controller can be chosen as

$$u(i,j) = \begin{bmatrix} -0.1940 & 0.0916 & -0.3894 \\ -0.3765 & 0.0497 & -0.5025 \end{bmatrix} x(i,j).$$

## V. Conclusions

This brief has addressed the problem of positive real control for uncertain 2-D discrete systems described by the FMLSS model. A version of positive realness for 2-D discrete systems has been established, which has been shown to be an extension of positive realness of 1-D discrete systems. A condition of the solvability of the above problem has been presented in terms of an LMI and the explicit formula of a desired state feedback controller has been given. The proposed control law guarantees both robust stability and extended positive realness of the closed-loop system with admissible parameter uncertainties.

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# Cyclostationary Noise in Radio-Frequency Communication Systems

Manolis T. Terrovitis, Kenneth S. Kundert, and Robert G. Meyer

***Abstract*—Because of the periodically time-varying nature of some circuit blocks of a communication system, such as the mixers, the noise which is generated and processed by the system has periodically time-varying statistics. An accurate evaluation of the system output noise is not straightforward as in the case where all the circuit blocks are linear-time-invariant and the noise that they generate is time-independent. We qualitatively examine here, conditions under which we can treat the noise at the output of every circuit block of a practical communication system as if it were time-invariant, in order to simplify the noise analysis without introducing significant inaccuracy in the noise characterization of the overall communication system.**

***Index Terms*—Cyclostationary noise, mixers, noise, time varying circuits.**

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## I. INTRODUCTION

The concepts of noise figure and noise temperature have been introduced to describe the noise performance of circuits and receivers [1], [2]. They are convenient performance metrics because the noise figure and noise temperature of a system of cascaded blocks can be found easily from the corresponding quantities of the individual blocks. However, the simple formulas for a system of cascaded blocks assume that the noise at the input and the output of every block is a wide-sense-stationary (WSS) process. The mixer is a very common block in modern receivers and is used to shift the frequency of an information signal by multiplying it by a signal at a different frequency, commonly supplied by a local oscillator (LO). There are two reasons why the mixer output noise is in fact not WSS but has periodically time-varying statistics. First, the operating points of the devices may vary with time, and second the transfer function of the noise signal from the point at which it is generated to the output can have time-varying characteristics [3]. The mixer output noise is a cyclostationary process and its complete description requires a periodically time-varying power-spectral density (PSD) $S(f, t)$ [4]. An accurate evaluation of the output noise when cyclostationary noise is processed by a linear periodically time-varying (LPTV) system is considerably more complicated than the evaluation of the output noise of a linear-time-invariant (LTI) system processing WSS noise. The corresponding analysis and methodology is given in [4], and a related circuit simulator has been presented in [5].

Despite the fact that the mixer output noise is cyclostationary, the noise figure calculated using the time-average output noise PSD has been traditionally used to characterize mixers, and the simple formulas for the noise figure of a system of cascaded blocks have been used to find the noise figure of a receiver. We shall show here that this treatment provides the correct noise characterization of a communication system in most practical cases, but we will examine cases in which it could lead to an inaccurate prediction. The pitfalls of applying the stationary process theory to cyclostationary signals have been presented in mathematical terms in [6]. Here, we discuss qualitatively, some related results that can be useful in the design of radio-frequency (RF) communication systems.

## II. CYCLOSTATIONARY NOISE AND ITS TIME AVERAGE

The complete description of a cyclostationary noise signal with its time-varying PSD $S(f, t)$, as opposed to its description with $S(f)$, the time-average of $S(f, t)$, is significant only when the block to which the cyclostationary noise is input is synchronized to the variation of $S(f, t)$ with time. This statement will be explained on an intuitive basis, and it also gains support from the following theorem [7].

If a uniformly distributed random variable from zero to one cycle period is added to the time variable $t$ of a cyclostationary process with PSD $S(f, t)$, (that is, the information about the phase of the periodically varying PSD is lost) the resulting process is stationary and its statistics are the time-average of the statistics of the cyclostationary process.

If the system to which the cyclostationary noise is input does not track the PSD variation with time, the phase of $S(f, t)$ for this system is unknown. In the absence of information about the phase of $S(f, t)$ the process becomes stationary, with PSD equal to the time-average of $S(f, t)$.

Usually, the noise performance of the analog part of a communication system consisting of a chain of RF circuit blocks, is characterized by measuring the time-average noise PSD at the output of the chain. Noise measuring equipment measures the noise PSD at a frequency $f$ by measuring the noise signal power at the output of a very narrow-band filter around $f$, without tracking the time variation of the noise statistics and provides the time-average PSD.

When a cyclostationary noise signal passes through a LTI filter and the time-average PSD is measured at the output, the same result is obtained if only the time-average PSD is considered at the input of the filter [4]. However, when a cyclostationary noise signal is fed to a time-varying system, consideration of only the time-average PSD of the input noise can lead in the general case to wrong results [4]. For instance, if the time-varying gain and the power of the input noise obtain their peak values simultaneously, considering only the time-average input noise will underestimate the output noise. The following example will help clarify the situation.

Consider that a WSS signal $n(t)$ with PSD $S_n(f)$ is fed to a mixer $A$, and the output of this $n_a(t)$ is fed to a mixer $B$, as shown in Fig. 1(a). The random signal $n(t)$ can represent noise present at the input of mixer $A$, or noise generated by its devices.[1] The mixing operation is modeled by multiplication of the input signal $n(t)$ with a periodic waveform (time-varying gain) generated by a local oscillator, $a(t)$ with frequency $f_{oa}$ and $b(t)$ with frequency $f_{ob}$, for mixers $A$ and $B$, respectively.[2] The output of mixer $A$ is a cyclostationary process whose time-average PSD consists of copies of $S_n(f)$ shifted in frequency integer multiple of $f_{oa}$, and weighted by different coefficients. It is easy to see that frequency components of $n_a(t)$ in distance integer multiple of $f_{oa}$ are correlated, since they contain the same frequency component of $n(t)$. A random process can be cyclostationary with cycle frequency $f_{oa}$ only if there exists correlation between two different frequency components in distance $f_{oa}$. The spectral correlation can be expressed in terms of the cyclic spectra, the Fourier components of the time-varying PSD, and in fact the $k$th cyclic spectrum for positive $k$ is the correlation between frequency components in distance $kf_{oa}$, while the $0$th order cyclic spectrum is the time-average PSD. A random process can be WSS only if any two different frequency components are uncorrelated [4]. The output of mixer $B$ is a cyclostationary process with two cycle frequencies $f_{oa}$ and $f_{ob}$. If $f_{oa}$ and $f_{ob}$ are commensurate (their ratio is a rational number), $n_b(t)$ can be viewed as cyclostationary with one cycle frequency equal to the maximum common divider frequency of $f_{oa}$ and $f_{ob}$.

### A. Effect of LO Frequency Relation

Let us examine now the spectral content of the output of mixer $B$ $n_b(t)$ at a frequency $f_{\rm out}$. Frequency components of $n_a(t)$ at frequencies $f_{\rm out} + kf_{ob}$, $k$ being an integer, are folded on $f_{\rm out}$ as shown in Fig. 1(b). If $nf_{oa} = mf_{ob}$ for some integers $n$ and $m$, there exists correlation among these components, and it is incorrect to add their power, as we would do if $n_a(t)$ were WSS, since a valid addition would require correlation terms. However, if the ratio of $f_{oa}$ and $f_{ob}$ is not a rational number, such integers $n$ and $m$ do not exist and simply adding the different frequency components of the time-average PSD $S_{na}(f)$ provides the correct result, since the added terms are uncorrelated.

In practice, the ratio of two LO frequencies generated by different free running oscillators can always be considered an irrational number, since because of the random phase error they cannot track each other.

[1]In the case of noise generated by devices with a time-varying operating point, this noise is cyclostationary and white, and its time variation can be incorporated to the system [3]. Therefore, in any case, the input noise $n(t)$ can be considered WSS. For every noise source inside the mixer, the time-varying gain is a different function.

[2]At high frequencies where reactive effects are not negligible, the mixing operation also depends on the input-signal frequency and is better modeled with a periodically-time-varying transfer function $A(f, t)$ [8], [3], instead of a periodically-time-varying gain $a(t)$. Frequency translation is described with the Fourier components of $A(f, t)$, the conversion transfer functions instead of the conversion gains. For simplicity, reactive effects are neglected in our mixer model. However, the qualitative arguments presented here also apply at high frequencies.

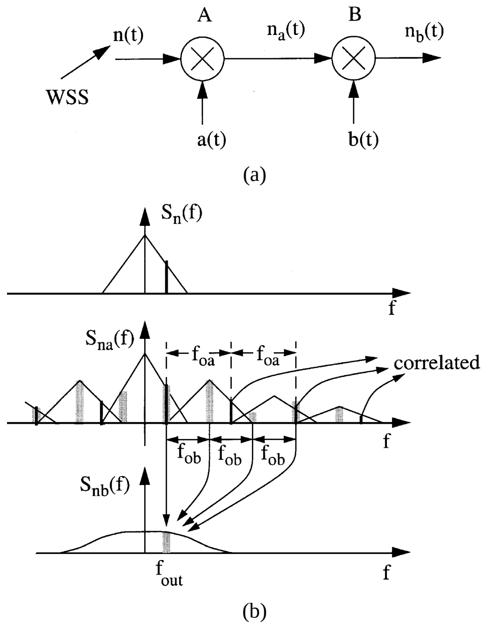


Fig. 1. (a) A cascade of two mixers. (b) Time-average PSD of noise at the input, after the first mixer and the output.

The situation is different however if the two LOs are locked to a common reference frequency. In a superheterodyne receiver which employs two mixers, it is a common practice to generate the two LO signals from two PLLs with a common reference frequency, which means that $f_{oa}/f_{ob}$ is a rational number $m/n$ (we will assume below that $m$ and $n$ are such that a common integer divider greater than one does not exist). Despite this, a rational frequency ratio $f_{oa}/f_{ob} = m/n$ with $m$ or $n$ very large numbers is expected to have the same practical effect as an irrational frequency ratio. In fact, the LO frequencies in a receiver chain are often chosen such that they do not have a simple relation in order to avoid spurious responses.

Assuming a smooth $b(t)$ with low frequency content, we can see that the conversion gain of mixer $B$ drops rapidly with the order of the sideband, and only the first few (for example up to 3 or 4) contribute significantly. Therefore, considering again the integers $m$ and $n$ that satisfy $f_{oa}/f_{ob} = m/n$, if $m$ is a large integer, in every set of correlated frequency components of $n_a(t)$ in distance integer multiple of $mf_{ob} = nf_{oa}$ that contribute to $f_{out}$, only one term contributes significantly and only a minor error is introduced by adding the power of all the components. If $n$ is large, assuming a smooth $a(t)$, the effect of noise correlation is also attenuated for a similar reason: the copy of $n(t)$ around $nf_{oa}$ has low power. Concluding, the effect of spectral correlation is insignificant if $a(t)$ is smooth and $n$ is large, or if $b(t)$ is smooth and $m$ is large, or both. Very often in practice, especially at high frequencies $a(t)$ and $b(t)$ are smooth functions, and unless the ratio of the two LO frequencies is a simple rational number $m/n$ with $m, n$ small integers, calculating the time-average at the first mixer output and treating it as if it were the PSD of WSS noise, does not introduce a significant error in the noise estimation at the second mixer output. Nevertheless, there are practical situations where the time-varying gain of a mixer is not a smooth waveform. An example is the sampling or subsampling mixer, in which case the time-varying gain is a pulse train which has high frequency content.

The above argument can be easily visualized in the time domain with the example of Fig. 2. Assume that $b(t)$ is an impulse train, so that mixer $B$ is essentially a sampling mixer as shown in Fig. 2(a) and that we desire to estimate the time-average power of the samples at the output of the sampler. Consider that the time-varying power $\sigma_a(t)$ of the cyclostationary noise $n_a(t)$—the integral of the time-varying PSD over all frequencies—at the first mixer output is the periodic function of time shown in Fig. 2(b). If $f_{oa} = f_{ob}$, or $f_{oa} = mf_{ob}$, we always sample $n_a(t)$ when $\sigma_a(t)$ is at the same point of the period as shown in Fig. 2(b), and if instead the time-average of $\sigma_a(t)$ is considered at the input of the sampler, we probably significantly overestimate or underestimate the output noise. In this case, since $b(t)$ is not a smooth function of time and its spectral content does not die out at high frequencies, the effect of spectral correlation is not diminished if $m$ is large. If $f_{oa}/f_{ob} = m/n$ is a rational number and $n$ is a small integer, we sample, repeatedly, only a few points in the period and it is possible that considering the time-average of $\sigma_a(t)$ at the sampler input will result again in an erroneous noise estimation. However, if $n$ is a large number, the same points of the period are repeatedly sampled, but they are many and uniformly distributed across a period, as shown in Fig. 2(c), so considering the time-average at the sampler input would give a practically correct result. When $f_{oa}/f_{ob}$ is not a rational number, after long enough time the whole period is uniformly sampled and in fact the same point is never sampled twice. In this case, time-averaging at the sampler input provides exactly the correct result.

Let us now examine the effect of the LO frequency relation in a more quantitative manner. Referring to Fig. 1, we can see that $n_b(t)$ consists of scaled copies of $n(t)$ shifted in frequencies $k_a f_{oa} + k_b f_{ob}$, where $k_a$ and $k_b$ are the sidebands at which the conversion gain of mixers $A$ and $B$, respectively, is significant, determined by the spectral content of the waveforms $a(t)$ and $b(t)$ and possibly as we shall see below by filtering the mixer outputs. If two of these frequencies coincide, the spectral correlation affects the output noise estimation. If $k'_a$ and $k'_b$ is a second set of mixer sidebands, the relation

$$k_a f_{oa} + k_b f_{ob} = k'_a f_{oa} + k'_b f_{ob} \tag{1}$$

or

$$\frac{k_b - k'_b}{k_a - k'_a} = -\frac{f_{oa}}{f_{ob}} \tag{2}$$

can only hold if $f_{oa}/f_{ob}$ is a rational number, as we also concluded before. Furthermore, if $f_{oa}/f_{ob} = m/n$, spectral correlation has an effect only if there are integers $k_a$, $k'_a$, $k_b$, and $k'_b$ that represent mixer sidebands with significant conversion gain such that

$$\frac{k_b - k'_b}{k_a - k'_a} = -\frac{m}{n}. \tag{3}$$

If for example, $a(t)$ and $b(t)$ are sinusoidal with frequencies $mf_o$ and $nf_o$, where $f_o$ is some reference frequency, $k_a$, $k'_a$, $k_b$, and $k'_b$ can only be $+1$ and $-1$, and spectral correlation can have an effect only if $n = m$.

Similarly, one can examine the effect of spectral correlation when a third mixer $C$ follows the chain of $A$ and $B$. Denoting the frequency of $C$ by $f_{oc}$ and the sidebands of $C$ with some significant conversion gain by $k_c$ and $k'_c$, spectral correlation affects the noise estimation only when there are mixer sidebands with significant conversion gain, such that

$$k_a f_{oa} + k_b f_{ob} + k_c f_{oc} = k'_a f_{oa} + k'_b f_{ob} + k'_c f_{oc}. \tag{4}$$

If the LO frequencies are related, i.e., $f_{oa} = mf_o$, $f_{ob} = nf_o$, $f_{oc} = pf_o$, where $f_o$ is some reference frequency and $m$, $n$, and $p$ integers with no common factors, (4) becomes

$$(k_a - k'_a)m + (k_b - k'_b)n + (k_c - k'_c)p = 0. \tag{5}$$

In this case, it is possible that conditions (4) and (5) hold for low-order sidebands, even if the LO frequency relation is not simple. For example, if $f_{oa} = 2000$ MHz, $f_{ob} = 660$ MHz and $f_{oc} = 10$ MHz the above equations are satisfied for $k_a - k'_a = 1$, $k_b - k'_b = -3$, and $k_c - k'_c = -2$.

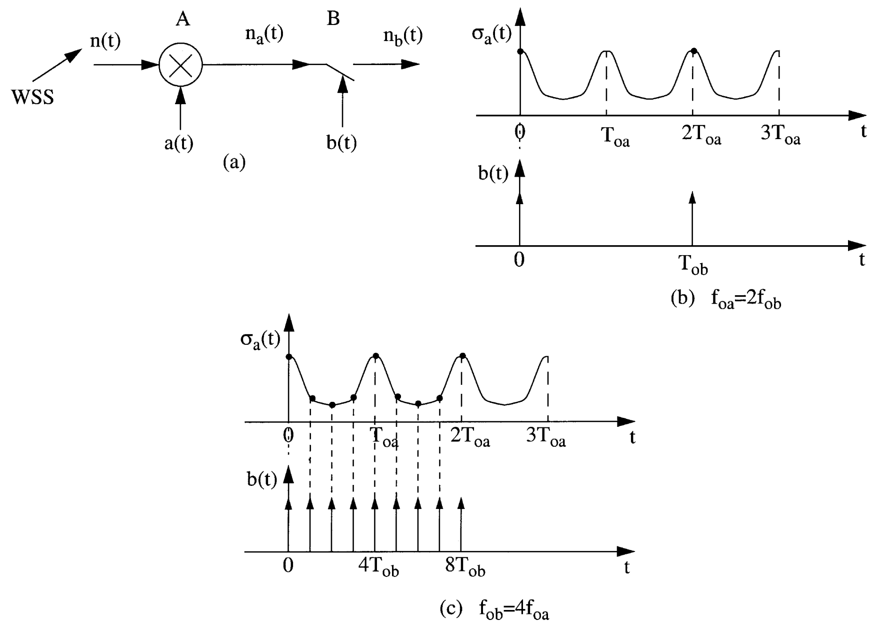


Fig. 2. Sampling cyclostationary noise.

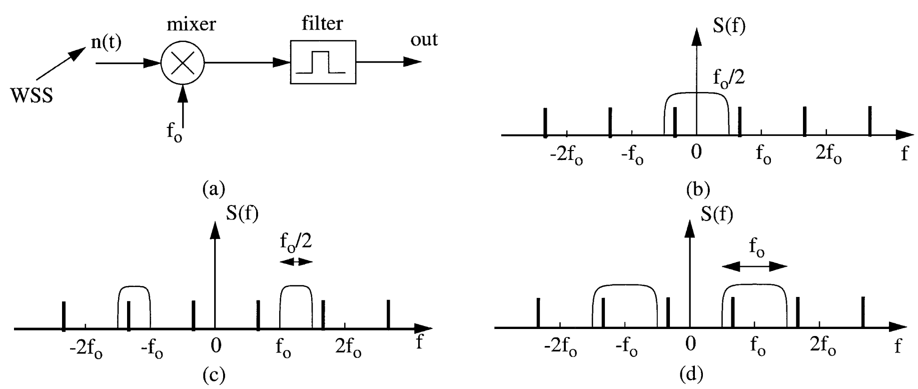


Fig. 3. Filtering cyclostationary noise.

### *B. Filtering a Cyclostationary Noise Process*

If filtering takes place at the output of a mixer, as in Fig. 3(a), it is possible that the noise at the output of the filter is stationary, and no cyclostationary noise considerations need to be made, or that the characteristics of the cyclostationary noise change. Some relevant theorems have been presented in [5], where they were derived in a mathematical way. Similar results can be found elsewhere [3], [9]. These theorems become intuitive by examining filtering of a set of correlated frequency components. Let us consider a cyclostationary noise process with cycle frequency $f_o$ and a set of correlated frequency components in distance integer multiple of $f_o$. The results of [5] can be observed as follows.

1) Consider a low-pass filter with cut-off frequency $f_o/2$ or lower, as in Fig. 3(b). One can see that only one component of the set of correlated components can fall in the window $[-f_o/2,\ f_o/2]$ that the filter allows to pass. Therefore any frequency components at the output of the filter are uncorrelated and the output noise is stationary.

2) Consider a single-sided bandpass filter, either upper band or lower band with respect to $f_o$, and bandwidth $f_o/2$ or less, as in Fig. 3(c). After filtering, only one frequency component of the correlated set remains, and the resulting noise is stationary.

3) Consider a bandpass filter with center frequency $f_o$ and bandwidth $f_o$ or less, as in Fig. 3(d). One can easily see that after filtering, the remaining correlated frequency components can only be in distance $2f_o$, and therefore only the stationary and the second-order cyclic spectra can exist.

Many other similar results can be visualized in a similar manner. For example if the filter is a low-pass filter with a cut off frequency $f_o$, the resulting process can contain only the stationary and first-order cyclic spectrum. A possible application of such a result as well as of result 3 above is the following. If it is known that the random signal at the output of mixer $A$ in Fig. 1 does not contain the $n$th order cyclic spectrum, $k_a - k'_a$ in (2)–(5) cannot be equal to $n$.

In a receiver chain the first mixer is typically followed by a bandpass IF filter. In this case one can apply the following theorem, which can also be verified easily by inspection.

If cyclostationary noise with cycle frequency $f_o$ passes through a bandpass filter with bandwidth $f_o/2$ or less, and the frequencies $k(f_o/2)$ where $k$ is an integer do not fall into the passband, the output noise is stationary.

The latter has been used in [3] but here we clearly define the necessary properties of the filter passband. Results 1 and 2 above can be seen as individual cases of this last theorem.

### *C. Mixing a Band-Limited Cyclostationary Noise Process*

In the previous section the passband characteristics of a filter following a mixer were related to the frequency of the LO waveform

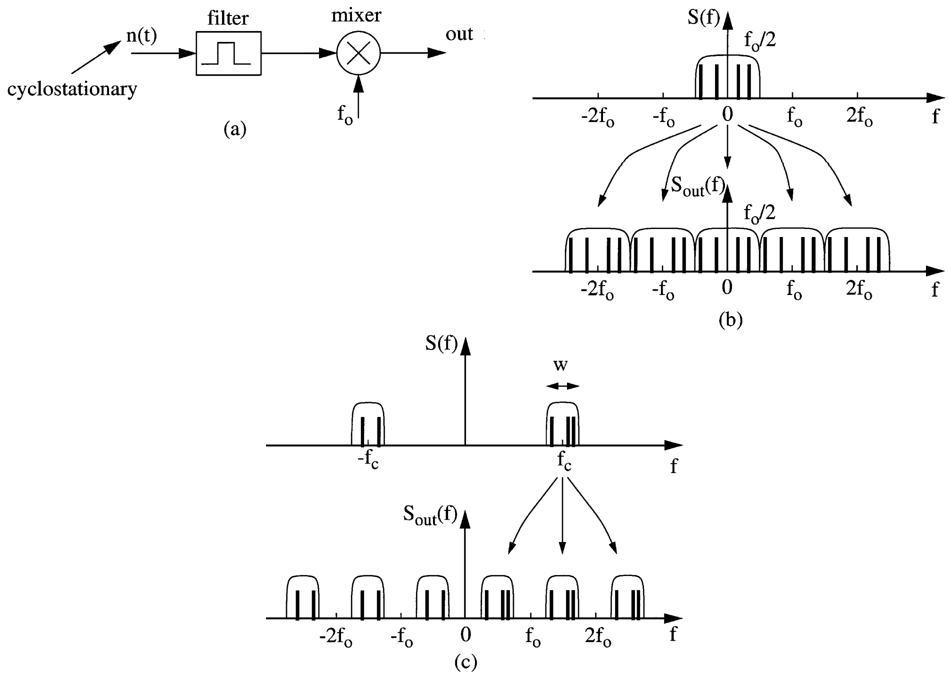


Fig. 4. Mixing band-limited cyclostationary noise.

driving the mixer in order for the output noise signal to have certain properties. Here, we will examine the case of Fig. 4(a) in which a general cyclostationary signal for which we have no information about the location of the correlated frequency components, passes through a filter and the filter output is fed to a mixer (or more generally a time-varying circuit). We will relate the filter characteristics with the frequency $f_o$ of the LO signal driving the mixer, in order for the time-average noise at the mixer output to be unaffected by the spectral correlation.

If the filter is low-pass with cut-off frequency $f_o/2$ or lower as shown in Fig. 4(b), no overlap will take place during mixing, and the average noise at the output will not be affected by spectral correlation. This situation appears often at the back-end of a receiver where sampling (for example performed by a switched capacitor filter or an analog to digital converter) is proceeded by an anti-alias filter.

If the filter is bandpass with center frequency $f_c$ and bandwidth $w$, as in Fig. 4(c), one can see that overlap will not happen if

$$|(k-k')f_o + 2f_o| > w \tag{6}$$

for all mixer sidebands $k$ and $k'$ with some significant conversion gain. This results from the observation that the positive passband will be transferred to frequency bands with center $kf_o + f_c$ and width $w$, the negative passband will be transferred to frequency bands with center $k'f_o - f_c$ and width $w$, and to avoid overlap the centers of the two frequency bands must be in distance greater than $w$.

## III. Two Cases Where Spectral Correlation is Significant

A practical situation that deserves attention is when an interfering signal or blocker is present at the input of a receiver. If this signal is strong it can change the operating point of the devices and affect the circuit noise performance. The noise generated by the circuit will acquire cyclostationary characteristics with cycle equal to the blocker period, and if the blocker is not filtered or modulated to a different frequency, it acts as a common LO for successive cascaded blocks. In this case, although a block can still be characterized with the noise figure under the presence of a blocker, use of the formulas for cascaded blocks to estimate the noise figure of the whole receiver can lead to an inaccurate prediction. This situation could arise for example when an in-band blocker is processed together with the weak desirable signal by the LNA and the RF mixer of a receiver.

Let us now consider noise introduced to a mixer from the LO port. The LO is a periodically time-varying circuit and it is possible that the noise at its output contains some cyclostationary component. The time-varying processing of this signal by the mixer tracks exactly the time variation of the noise statistics since the mixer instantaneous operating point is determined by the LO drive. Therefore, it is not correct to time-average the noise PSD at the LO output and use it as if it were a WSS process.

## IV. Conclusion

We qualitatively examined the significance of the cyclostationary nature of the noise generated in a communication system. We saw that cyclostationarity is equivalent to the presence of correlated components in the frequency spectrum. From the above discussion it results that in the majority of the practical cases, use of the concept of noise figure and considering only the time-average component of the cyclostationary noise at the input and the output of every block does not introduce significant inaccuracy in the noise characterization of the overall system for two reasons. First, the local oscillator frequencies used usually do not have a simple relation and the situation resembles the case at which the two frequencies are noncommensurate. Second, usually filtering takes place in several places in the receiver chain which converts the cyclostationary noise to stationary noise. However, we examined practical cases where cyclostationarity cannot be ignored, namely when the subsequent stage is time-varying synchronously with the cyclostationarity, as for example when the subsequent stage is driven nonlinearly by the stage generating cyclostationary noise. In these situations, if noise characterization is desirable by means of a circuit simulator which provides the time-average output noise the time-varying circuit blocks must be simulated together. Alternatively, the simulator of [5] can be used to calculate the cyclic spectra of every block separately and create appropriate macro-models which can then be used in a behavioral level simulation.

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# Optimal Wire-Sizing Function Under the Elmore Delay Model With Bounded Wire Sizes

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***Abstract*—In this brief, we develop the optimal wire-sizing functions under the Elmore delay model with bounded wire sizes. Given a wire segment of length $\mathcal{L}$, let $f(x)$ be the width of the wire at position $x$, $0 \leq x \leq \mathcal{L}$. We show that the optimal wire-sizing function that minimizes the Elmore delay through the wire is $f(x) = ae^{-bx}$, where $a > 0$ and $b > 0$ are constants that can be computed in $O(1)$ time. In the case where lower bound ($L > 0$) and upper bound ($U > 0$) of the wire widths are given, we show that the optimal wire-sizing function $f(x)$ is a truncated version of $ae^{-bx}$ that can also be determined in $O(1)$ time. Our wire-sizing formula can be iteratively applied to optimally size the wire segments in a routing tree.**

***Index Terms*—Elmore delay, optimal, wire sizing.**

## I. INTRODUCTION

As very large-scale integration (VLSI) technology continues to scale down, interconnect delay has become the dominant factor in deep submicron designs. As a result, wire sizing plays an important role in achieving desirable circuit performance. Recently, many wire-sizing algorithms have been reported in the literature [1]–[5]. All these algorithms size each wire segment uniformly, i.e., identical width at every position on the wire. In order to achieve nonuniform wire sizing, existing algorithms have to chop wire segments into large number of small segments. Consequently, the number of variables in the optimization problem is increased substantially and thus results in long runtime and large storage.

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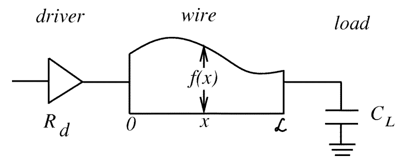


Fig. 1. Six types of optimal wire-sizing functions.

In [6], the optimal wire shape with minimal Elmore delay without wire-size constraints are presented using the calculus of variation methods. In this brief, we develop the optimal wire-sizing function for minimal Elmore delay with the wire-size constraints using only basic mathematical methods. Given a wire segment $W$ of length $\mathcal{L}$, a source with driver resistance $R_d$, and a sink with load capacitance $C_L$. For each $x \in [0, \mathcal{L}]$, let $f(x)$ be the wire width of $W$ at position $x$. Fig. 1 shows an example. Let $r_0$ and $c_0$ be the respective wire resistance and wire capacitance per unit square. Let $D$ be the Elmore delay from the source to the sink of $W$. We show that the optimal wire-sizing function $f$ that minimizes $D$ satisfies a differential equation which can be analytically solved. We have $f(x) = ae^{-bx}$, where $a > 0$ and $b > 0$ are constants that can be computed in $O(1)$ time. These constants depend on $R_d$, $C_L$, $\mathcal{L}$, $r_0$, and $c_0$. Our method is extended to solve the case where lower bound ($L > 0$) and upper bound ($U > 0$) on the wire widths are given, i.e., $L \leq f(x) \leq U$, $0 \leq x \leq \mathcal{L}$, we show that the optimal wire-sizing function $f(x)$ is a truncated version of $ae^{-bx}$ which can also be determined in $O(1)$ time. Our wire-sizing formula can be iteratively applied to optimally size the wire segments in a routing tree.

The remainder of this brief is organized as follows. In Section II, we show how to compute the Elmore delay for nonuniformly sized wire segments. In Section III-A, we derive the optimal wire-sizing function when the wire widths are not constrained by any bounds. In Section III-B, we consider the case where lower and upper bounds for the wire widths are given. We discuss the importance of our wire-sizing formula in sizing the wire segments in a routing tree in Section IV. Finally, we present some experimental results and concluding remarks in Section V.

## II. ELMORE DELAY MODEL

We use the Elmore delay model [7]. Suppose $W$ is partitioned into $n$ equal-length wire segments, each of length $\triangle x = \mathcal{L}/n$. Let $x_i$ be $i\triangle x$, $1 \leq i \leq n$. The capacitance and resistance of a wire segment $i$ can be approximated by $c_0 \triangle x f(x_i)$ and $r_0 \triangle x / f(x_i)$, respectively. Thus, the Elmore delay through $W$ can be approximated by

$$D_n = R_d \left( C_L + \sum_{i=1}^{n} c_0 f(x_i) \triangle x \right) + \sum_{i=1}^{n} \frac{r_0 \triangle x}{f(x_i)} \left( \sum_{j=i}^{n} c_0 f(x_j) \triangle x + C_L \right). \quad (1)$$

The first term is the delay of the driver, which is given by the driver resistance $R_d$ multiplied by the total capacitance of $W$ and $C_L$. The second term is the sum of the delay in each wire segment $i$, which is given by its own resistance $r_0 \triangle x / f(x_i)$ multiplied by its downstream capacitance $\sum_{j=i}^{n} c_0 f(x_j) \triangle x + C_L$. (See Fig. 2.) As $n \to \infty$, $D_n \to D$ where

$$D = R_d \left( C_L + \int_0^{\mathcal{L}} c_0 f(x)\, dx \right) + \int_0^{\mathcal{L}} \frac{r_0}{f(x)} \left( \int_x^{\mathcal{L}} c_0 f(t)\, dt + C_L \right) dx \quad (2)$$

is the Elmore delay through the driver and $W$.