Simulation and Modeling of Intermodulation Distortion in Communication Circuits

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***Abstract* — Distortion in circuits along the signal path of a transceiver plays a key role in determining the overall performance of digital communication systems. This paper describes how recent improvements in the mixed frequency/time algorithm (MFT) expand our ability to predict the distortion of these circuits. The MFT algorithm extends traditional shooting methods to directly compute the quasiperiodic steady-state response of circuits driven by two or more periodic signals, each at independent frequencies.**

With these improvements, MFT can be directly applied to accurately and efficiently compute the intermodulation distortion of large circuits driven by a small number of discrete tones. This is representative of the signals found in receivers. However in transmitters, concern focuses on the affect of intermodulation distortion from complex digitally modulated signals. Such signals cannot be handled directly by MFT, but we demonstrate how a macromodel can be constructed that is used to rapidly predict the spectral regrowth caused by the power amplifier and associated circuitry. This approach allows much more of the transmitter to be simulated than with envelope-transient methods.

I. Introduction

The increasing demand for low-cost mobile communication systems has greatly expanded the need for simulation algorithms that are both efficient and accurate when applied to RF communication circuits. These circuits are a special challenge to simulate because they process signals that consist of a high frequency carrier and a low frequency modulation. Typically the carrier frequency ranges from 1-5 GHz and the modulation from 10 kHz to 1 MHz.

Often the actual modulated carrier signals are modeled for the purposes of simulation by simpler quasiperiodic signals, where a quasiperiodic signal is periodic carrier modulated with one or more periodic signals. Quasiperiodic signals result when a nonlinear circuit is driven with two or more periodic signals at unrelated frequencies. Currently, harmonic balance is the most commonly used method for computing the quasiperiodic response of a circuit [4,5], however it suffers from accuracy and efficiency problems when signals contain abrupt transitions. Unfortunately, this is the case with most communication circuits. For example, mixers are always driven with a square wave LO with rapid transitions because it results in higher conversion gain and better noise performance. In addition, circuits such as sample/holds and switched-capacitor filters are driven with a binary-valued clock signal. The mixed-frequency/time method (MFT) is an promising alternative to harmonic balance because it is capable of both accurately and efficiently simulating circuits where one of the periodic components of a quasiperiodic signal exhibits abrupt transitions.

II. The Mixed Frequency-Time Method

The goal of the MFT algorithm [3,4] is to find a quasiperiodic steady state solution to the equations that govern the operation of the circuit. Quasiperiodic steady state means that the signals in the circuit can be represented using the harmonics of a finite number of fundamental frequencies, for example,

$$v(t) = \sum_k \sum_l V_{kl} e^{j2\pi(lf_0 + kf_1)t} , \qquad (1)$$

where, for simplicity, we limit the number of fundamental frequencies to two, f_0 and f_1. Consider sampling the signal $v(t)$ at one of the fundamental frequencies, f_0, which is referred to as the clock. The sampled signal is related to the continuous signal by $\hat{v}_n = v(nT_0)$, where $T_0 = 1/f_0$.

The MFT method works by computing the discrete sequence $\hat{v}$ instead of the continuous waveform v. First we define the state transition function, $\phi(v_0, t_0, t_1)$, to be the solution of the circuit equations at t_1 given that it starts from the initial condition v_0 at t_0. If we require that every $\hat{v}_n$ is related to the subsequent sample point $\hat{v}_{n+1}$ by

$$\hat{v}_{n+1} = \phi(\hat{v}_n, nT_0, (n+1)T_0) , \qquad (2)$$

then all the $\hat{v}_n$ will satisfy the circuit equations.

The transition function in (2) can be computed by standard circuit transient analysis and serves to translate between the continuous signal and the discrete representation. The key to the MFT method is to require that the samples $\hat{v}_n$ represent a quasiperiodic signal. This requirement is easily enforced because, as shown in Figure 1, sampling a 2-fundamental quasiperiodic signal at one of the fundamental frequencies results in a sampled waveform being 1-fundamental quasiperiodic, or simply periodic. In other words,

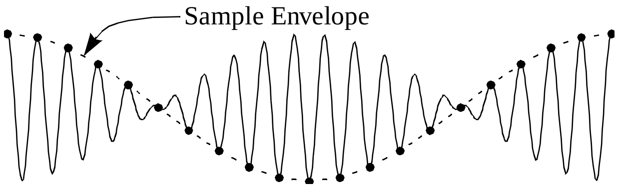

Fig. 1. The sample envelope is the waveform that results from sampling a signal at a rate equal to that of the clock.

the sampled waveform can be written as a Fourier series with the clock tone removed,

$$\hat{v}_n = v(nT_0) = \sum_{k=-\infty}^{\infty} \hat{V}_k e^{j2\pi knf_1T_0} . \quad (3)$$

Alternatively, one can write

$$\hat{v} = F^{-1}\hat{V} , \quad (4)$$

which states that $\hat{v}$ is the inverse Fourier transform of $\hat{V}$. Consider the n^{th} sample interval and let $x_n = \hat{v}_n$ be the solution at the start of the interval and $y_n = \hat{v}_{n+1} = x_{n+1}$ be the solution at the end. Then, (2) uses the circuit equations to relate the solution at both ends of the interval,

$$y_n = \phi(x_n, nT_0, (n+1)T_0) . \quad (5)$$

Define Φ as the function that maps the sequence x to the sequence y by repeated application of (5),

$$y = \Phi(x). \quad (6)$$

Let $X = Fx$ and $Y = Fy$ (X and Y are the Fourier transforms of x and y). Then, from (3) and since $y_n = x_{n+1}$,

$$X_k = e^{-j2\pi kf_1T_0}Y_k , \quad (7)$$

or

$$X = D_{T_0}Y , \quad (8)$$

where D_{T_0} is a diagonal delay matrix with $e^{-j2\pi kf_1T_0}$ being the k^{th} diagonal element, which is written in the time domain as

$$x = F^{-1}D_{T_0}Fy , \quad (9)$$

Together, (6) and (9) make up the MFT method and can be combined into

$$x = F^{-1}D_{T_0}F\Phi(x) \text{, or} \quad (10)$$

$$\hat{v} = F^{-1}D_{T_0}F\Phi(\hat{v}) . \quad (11)$$

Equation (11)is a implicit nonlinear equation that can be solved for $\hat{v}$.

In practice, the signals in the circuits are band-limited, and so only a finite number of harmonics is needed. Thus the envelope shown in Figure 1 can be completely specified by only a few of the sample points $\hat{v}_n$. With only K harmonics needed, (2) is evaluated over $2K+1$ distinct intervals. In particular, if the circuit is driven with one large high frequency signal at f_0, which is referred to as the clock, and one moderately sized sinusoid at f_1, then the number of harmonics needed, K, is small and the method is efficient. The total simulation time is proportional to the number of harmonics needed to represent the sampled waveform and is independent of the period of the low-frequency beat tone or the harmonics needed to represent the clock signal.

Equation 11 serves to relate the starting and ending points of the solution of the circuit equations over each interval. Shooting methods are the most common method for solving such boundary-value problems. They use transient analysis to solve the circuit equations over an interval, which brings two important benefits. First, transient analysis has a natural ability to efficiently handle abruptly discontinuous signals because the timestep shrinks to follow rapid transitions. Second, it easily handles the strongly nonlinear behavior of the circuit as it responds to the large clock signal.

III. Improvements to the MFT Method

Several improvements to the MFT method have been developed recently, which make it able to handle large nonlinear circuits accurately and reliably [1]. The first improvement involves the selection of the intervals computed by the MFT algorithm. These intervals need to be equally spaced over the period of the lowest beat tone. In the original version of the algorithm, the intervals were equally spaced in a statistical sense and a special almost-periodic Fourier transform or APFT was used. Use of the APFT overcame two difficult issues: the sample rate and the beat frequency being noncommensurate, and the sample envelope occasionally being quasiperiodic. The somewhat random placement of the intervals generated a level of error that was quite noticeable in the final results. In the latest version, these issues are instead overcome by using a multidimensional Fourier transform. The resulting intervals are precisely evenly spaced and the error associated with the APFT is eliminated.

The second improvement is the application of Krylov subspace methods. The MFT equation (11) is solved using Newton's method, which is an iterative procedure that generates a series of linear equations. The equations involve the Jacobian of the state transition function, which is a dense $N \times N$ matrix, where N is the number of circuit equations. In the past, these equations were solved using Gaussian elimination; the time required is proportional to N^3, which results in this method becoming prohibitively expensive when applied to large circuits. In the latest implementation of the MFT algorithm, Krylov subspace methods replace Gaussian elimination, and the time required to solve the equations becomes proportional to N rather than N^3. As a result, MFT is now able to handle much larger circuits than before.

In addition to the efficiency and accuracy advantages already described, the new MFT method is very suitable for parallel computation since the transient integrations of the $2K+1$ intervals can be done independently, as can the initial preconditioning of the linear equations that is required by the Krylov subspace methods. Another advantage of the new MFT method is that it has an efficient out-of-core implementation since its data access pattern is very local and sequential.

IV. Simulation of Communication Circuits

This section shows two applications of the new MFT method. As common characteristics, each circuit possesses a large binary clock or LO as well as a high clock to signal frequency ratio.

The first example is a high-performance image rejection receiver. It consists of a low-noise amplifier, a splitting network, two double-balanced mixers, and two broadband Hilbert transform output filters combined with a summing network that is used to suppress the undesired side-band. A limiter in the LO path is used for controlling the amplitude of the LO. Since the LO is a large square wave, it is difficult to simulate accurately with harmonic balance. It is a rather large RF circuit that contains 167 bipolar transistors and uses 378 nodes. This circuit generated 987 equations in the simulator.

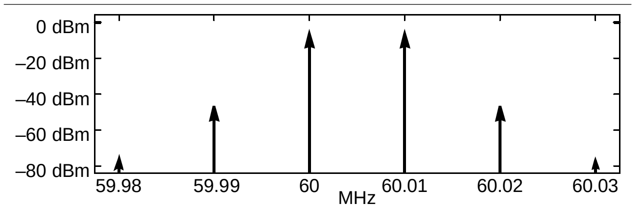

Fig. 2. Intermodulation distortion of a receiver.

To determine the intermodulation distortion characteristics, the circuit was driven by a 780MHz LO and two 50mV closely placed RF inputs, at 840MHz and 840MHz+10KHz, respectively. Three harmonics were used to model each of the RF signals. 200 time points were used in each transient clock-cycle integration, considered to be conservative in terms of accuracy for this circuit. As a consequence, $987 \times (2 \times 3+1)^2 \times 200 = 9{,}672{,}600$, or around ten million unknowns were generated. It took 55 CPU minutes to finish on a Sun UltraSparc10 workstation with 128 MB of physical memory and a 300MHz CPU clock. Figure 2 shows 3rd and 5th order distortion products.

To understand the efficiency of the MFT method, consider that traditional transient analysis would need at least 80,000 cycles of the LO to compute the distortion, a simulation time of over two days.

The second example is a low-pass switched-capacitor filter of 4kHz bandwidth and having 238 nodes, resulting in 337 equations. To analyze this circuit, the MFT analysis was performed with an 8-phase 100kHz clock and a 1V sinusoidal input at 100Hz. The 1000 to 1 clock to signal ratio makes this circuit difficult for traditional circuit simulators to analyze. In the MFT method, five harmonics were used to model the input signal. The eight-phase clock resulted in the need to use about 1250 timepoints in each transient integration. This brings the total number of variables solved by the analysis to $337 \times (2 \times 5+1) \times 1250 = 4{,}633{,}750$, near six million. The simulation took a little less than 30 minutes CPU time to finish, on a Sun UltraSparc1 workstation with 128 MB memory and a 167MHz CPU clock. Figure 3 shows the output spectrum of the filter.

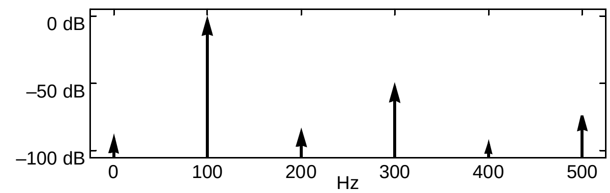

Fig. 3. Harmonic distortion of a switched-capacitor filter.

V. Predicting Spectral Regrowth of a Transmitter

A typical digital transmitter architecture is shown in Figure 4. The low-pass filters can eliminate out-of-band signal components from the input baseband signal, but these components may re-appear as a result of intermodulation distortion generated within the transmitter, for example in the downstream power amplifier. This tendency to regain undesirable signal components is called "spectral regrowth." Adjacent Channel Power Ratio (ACPR) is a popular measure of spectral regrowth in digital transmitters and is often a design specification. To get the frequency resolution required for accurate estimation of ACPR in Code Division Multiple Access (CDMA) systems, the simulation must often process a random sequence of between 500 to 10k input symbols. Even with envelope techniques [6], direct transistor-level simulation requires a taxing amount of resources except for simple circuits. However, the MFT method enables fast indirect ACPR estimation.

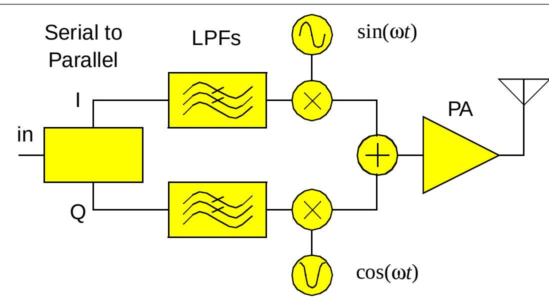

Fig. 4. A digital direct conversion transmitter.

The MFT algorithm is first used to extract a behavioral baseband-equivalent model of the transmitter. In transmitter circuits the input baseband signal is usually well within the transmitter's bandwidth, so a memoryless model often suffices. Because the behavioral model abstracts away the carrier and unnecessary circuit details, the following ACPR calculation step is fast regardless of circuit size or complexity.

The most familiar spectral regrowth mechanism is intermodulation distortion of amplitude modulation (AM) signal components entering the power amplifier. Modulation schemes that carrier information only on the carrier phase or frequency try to minimize spectral regrowth by eliminating this AM component. Even so, the digital baseband filters can convert discontinuous phase changes into amplitude transients in the composite RF signal. Models based on AM/AM and AM/PM conversion [2] capture most such power amplifier related distortion mechanisms. However, imperfections in the *I/Q* modulators can convert input phase variations into output amplitude and phase variations that also contribute to the distortion. For this reason we also include PM/AM and PM/PM conversion effects in our model.

In extracting the behavioral model using the MFT algorithm, the inputs are best written in polar coordinates, i.e., magnitude ρ and phase ϕ. The mapping $f(\rho,\phi)$ from input to output is periodic in ϕ and so can be expressed as a Fourier series,

$$f(\rho, \phi) = \sum_k A_k(\rho) e^{jk\phi} , \tag{12}$$

where the magnitude-dependent Fourier coefficients are

$$A_k(\rho) = \frac{1}{2\pi}\int_0^{2\pi} f(\rho, \phi) e^{-jk\phi} d\phi . \tag{13}$$

To extract the Fourier coefficients, the *I* and *Q* inputs of the transmitter are driven with sinusoids in quadrature, at a frequency within the transmitter's bandwidth. For these circular input trajectories, $\phi = \omega_0 t$, and the Fourier coefficients are given by

$$A_k(\rho) = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} f(\rho, \omega_0 t) e^{-jk\omega_0 t} dt . \tag{14}$$

Thus for a given input magnitude, the Fourier coefficients are obtained directly from the output spectrum calculated by the MFT. For example if the input circle is large enough to alternately saturate the modulators, the –3 harmonic of the complex baseband tone dominates the distortion. Let the carrier frequency be ω_c. The output spectrum has lines at $\omega_c+\omega_0$ and $\omega_c - 3\omega_0$. The MFT computes the real and imaginary parts of the Fourier coefficients. The simulation is repeated for a range of input magnitudes to capture the magnitude dependence of the Fourier coefficients. At each input magnitude, the fundamental and its relevant harmonics are recorded for interpolation. The model implementation reads the recorded data then processes any amount of input baseband data according to (12).

Figure 5 shows a simulated ACPR measurement. The circuit included the up-conversion image-reject mixer and the power amplifier and consists of 46 transistors generating 328 equations in the circuit simulator. A 7^{th} order model took 2.5 hours to extract and simulated 30ms of CDMA data (40k chips @ 1.25Mchips/sec) in less than 4 minuess using a dataflow simulator and a Sun Ultra 1.

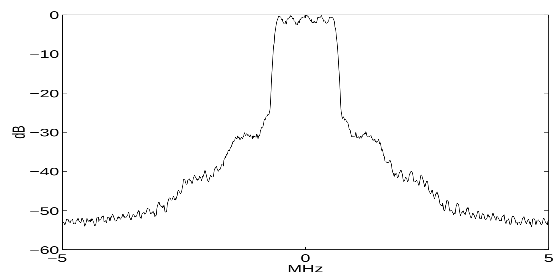

Fig. 5. Output of the model.

VI. Conclusion

Improvements to the MFT method make it possible to efficiently compute the intermodulation distortion of common communication circuits. There is no reasonable alternative on those circuits driven by a large clock, such as mixers, switched-capacitor filters, sample/holds, etc. In addition, a macromodel is presented that is used for predicting ACPR of transmitters. This macromodel is efficiently extracted using the MFT method and allows ACPR to be predicted for much more of the transmitter than is possible with other available methods.

References

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